## Applicative structure in vector space models

Márton Makrai，Dávid Nemeskey，András Korna
HAS Computer and Automation Research Institute

The problem
Mikolov［3］suggests
king - queen $=$ male - female
By commutativity
king - male $=$ queen - female $=$＇ruler，gender unspecified＇ But with function application

Victoria $=$ queen $\oslash$ England and
Victor $=$ king $\oslash$ Italy
If the function application operator $\oslash$ is simply another vector to be added to the representation，the same logic would yield that Italy is the male counterpart of female England

Overview
We introduce a new 100 －dimensional embedding obtained by spectral clustering of a graph describing the conceptual struc ture of the lexicon．We use the embedding directly to investi－ gate sets of antonymic pairs，and indirectly to solve the prob lem outlined above by treating $\varnothing$ and $\theta$ not as a vectors but as transformations．

Lexical decomposition
The standard model of lexical decomposition［2］divides lexical meaning in a systematic component，given by a tree of（gen－ erally binary）features，and an accidental component they call the distinguisher．


Antonymic pair lists


For a set of male and female words，such as〈king，queen＞，〈uncle，aunt＞，〈actor，actress〉，etc．，the dif ference between words in each pair should represent the idea of gender．Similarly for pairs differing in some other feature To test the hypothesis，we associated antonymic word pairs $\left\langle x_{i}, y_{i}\right.$ from WordNet 4 to 26 classes e．g．End／beginning GOOD／BAD，

| GOOD |  | vertical |  |
| :---: | :---: | :---: | :---: |
| safe | out | raise | level |
| peace | war | tall | short |
| pleasure | pain | rise | fall |
| ipe | green | north | south |
| efend | attack | shallow | eep |
| affirmative | negative | superficial | profou |

Table 1：Word pairs associated to features GOOD and vertical

## Test

For $k$ pairs $\mathbf{x}_{i}, \mathbf{y}_{i}$ we are looking for a common vector $\mathbf{a}$ such that

$$
\mathrm{x}_{\mathbf{i}}-\mathrm{y}_{i} \approx \mathbf{a}
$$

－Find $\operatorname{argmin}_{\mathbf{a}}$ Err

$$
E r r=\sum_{i}\left\|\mathbf{x}_{\mathbf{i}}-\mathbf{y}_{i}-\mathbf{a}\right\|^{2}
$$

－ $\operatorname{argmin}_{\mathbf{a}} E r r$ is actually the arithmetic mean of the vectors $\mathbf{x}_{\mathbf{i}}-\mathbf{y}_{i}$
Is the minimal Err any better than what we could expect from a bunch of random $\mathbf{x}_{\mathbf{i}}$ and $\mathbf{y}_{i}$ ？
100 random pairings of the words to estimate the erro distribution，computing the minima of

$$
E r r_{\text {rand }}=\sum_{i}\left\|\mathbf{x}_{\mathbf{i}}^{\prime}-\mathbf{y}_{i}^{\prime}-\mathbf{a}\right\|^{2}
$$

－Is the error of the correct pairing，Err at least 2 or 3
standard deviations $(\sigma)$ away from the mean of $E r r_{r a n d}$ ？
－features above the first line $\rightarrow$ antonymic relations are wel captured by the embeddings
－features below the second line $\rightarrow$ antonymic relations are not captured by the embeddings
－caused by size？
Embedding based on conceptual representation

## －Input：a graph

nodes are concepts
$A \rightarrow B$ iff $B$ is used in the definition of $A$
－base vectors are obtained by the spectral clustering method pioneered by［6］：
the incidence matrix of the conceptual network is replaced by a affinity matrix whose $i j$－th element is formed by computing the
－the first few（in our case，100）eigenvectors are used as a basis．
a word $w_{i}$ in the basic vocabulary is included in the graph and corresponds to a base vector $b_{i}$
for other words $w$ in the dictionary，we take the definition of any word $w$ in the Longman Dictionary of Contemporary English，we form $V(w)$ as the sum of the $b_{i}$ for the $w_{i}$ s that appeared in the definition of $w$（with multiplicity）
－stopwords：the 19 most frequent words

Results with embeddings

|  | HLBL［5］original |  |  | HLBL scaled |  |  |  | SENNA［1］ |  |  |  |  | 4 llang |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| pairs name | Err $\quad m$ |  |  | Err |  |  |  |  | Err |  | $\sigma$ |  | Er | $m$ | $\sigma$ |  |
| 32 many | 40.565 .8 | 2.69 | 9.39 |  | 565.8 | 2.82 | 8.98 |  |  |  | 9.4 | 10.5 | 0.6 | 0.78 | 0.07 | 2.11 |
| 42 vertical | 69.199 .1 | 3.43 | 8.74 | 69.1 | 198.9 | 3.58 | 8.34 |  | 1．38e＋ | $2.94 \mathrm{e}+03$ | 122 |  | 0.808 | 1.69 | 0.203 | 4.34 |
| 156 good | 254301 | 6.74 | 6.96 | 254 | 4302 | 6.19 | 7.79 |  | 6．47e＋03 | $1.05 \mathrm{e}+04$ | 229 | 7.5 | 3.78 | 38 | 0.186 | 26 |
| 49 in | 69.794 .9 | 4.27 | 5.92 | 69.7 | 794.5 | 4.35 | 5.7 |  | 1．71e＋03 | 3．55e＋03 | 128 |  | 1.13 | 1.63 | 0.137 | 3.68 |
| 48 same | 93.8112 | 3.29 | 5.48 | 93.8 | 8113 | 3.11 | 6.04 |  | 2．11e＋03 | $3.53 \mathrm{e}+03$ | 120 | 11.8 | 1.39 | 1.71 | 0.149 | 2.09 |
| 20 progress | 21.528 .5 | 1.56 | 4.45 | 21.5 | 528.7 | 1.44 |  |  | 801 | 1．37e＋03 | 86.2 | 6.62 | 0.432 | 0.67 | 0.067 | 3.5 |
| 28 end | 35.351 .8 | 3.75 | 4.41 | 35.3 | 352.8 | 3.67 | 4.79 |  | 798 | $1.78 \mathrm{e}+03$ | 137 | 7.14 | 0.748 | 3.6 | 0.539 | 5.3 |
| 12 | 10.814 .6 | 0.978 | 3.9 | 10.8 | 4.8 | 1.09 | 3.67 |  | 461 | 70 | 72. | 3.4 | 0. | 0.15 | 0.049 | ． 319 |
| 18 mental | 31.736 .2 | 1.31 | 3.45 | 31.7 | 736.3 | 1.14 | 4.08 |  | 830 | $1.2 \mathrm{e}+03$ | 57.4 | 6.4 | 0.60 | 0.694 | 0.059 | 1.49 |
| 65 active | 95.2112 | 5.19 | 3.32 | 95.2 | 2113 | 5.36 | 3.32 |  | $2.51 \mathrm{e}+03$ | 4．07e＋03 | 196 |  | 1.75 | 1.95 | 0.142 | 1.45 |
| 36 time | 59.270 .4 | 3.43 | 3.26 | 59.2 | 270 | 3.42 | 3.16 |  | 1．49e＋03 | $2.36 \mathrm{e}+03$ | 113 |  | 0.845 | 1.46 | 0.175 | 35 |
| 32 sophis | 65.674 .7 | 2.84 | 3.21 | 65.6 | 675.4 | 2.86 | 3.42 |  | $1.26 \mathrm{e}+03$ | $2.25 \mathrm{e}+03$ | 93.3 |  | 0.86 | 0.988 | 0.106 | 1.17 |
| 23 whole | 39.345 .1 | 1.87 | 3.14 | 39.3 | 345.4 | 91 | 21 |  | $1.06 \mathrm{e}+03$ | $1.65 \mathrm{e}+03$ | 84.1 | 7.07 | 0.70 | 1.4 | 0.2 | 3.19 |
| 34 y | 62.170 .8 | 3.45 | 2.52 | 62.1 | 7.6 | 84 | 22 |  | 4e＋03 | 2．29e＋03 | 122 |  | 0 | 0.703 | 0.13 | 2.89 |
| 12 front | 11.916 .5 | 2.15 | 2.14 | 11.9 | 916.1 | 2.25 | 1.87 |  | 371 | 635 | 73.8 | 3.58 | 0.20 | 0.26 | 0.0539 | 1.1 |
| 8 single | 7.8510 .4 | 1.31 | 1.94 | 7.85 | 7． 10.4 | 1.54 | 1.64 |  | 282 | 529 | 56.1 | 4.41 | 0.107 | 0.166 | 0.0516 | 1.15 |
| 14 primary | 24.428 .1 | 2.15 | 1.74 | 24 | 28.4 | 1.99 |  |  | 713 | 1．01e＋03 | 85.3 | 3．47 | 0.54 | 0.505 | 0.0583 | ． 718 |
| 14 gender | 15.318 .3 | 1.88 | 1.62 | 15.3 | 318.3 | 1.74 | 1.73 |  | 258 | 655 | 70.6 |  | 0.5 | 2.51 | 0.497 | 4.04 |
| 8 sound | 11.612 .7 | 0.744 | 1.52 | 11.6 | 612.7 | 0.833 | 1.32 |  | 324 |  |  |  | 0.138 | 0.142 | 0.0397 | 0.112 |
| 16 know | 25.127 .2 | 1.83 | 1.18 | 25.1 | 127.2 | 1.93 | 1.09 |  | 714 | 1．04e＋03 | 65.3 |  | 0.435 | 0.611 | 0.0766 | 2.29 |
| 10 angular | 18.816 .3 | 2.19 | 1.14 | 18.8 | 816.3 | 2.03 | 1.22 |  | 371 | 457 | 49.9 | 1.73 | 0.15 | 0.16 | 0.028 | 0757 |
| 10 real | 1313.9 | 1.09 | 0.808 | 13 | 314 |  | 0．844 |  | 442 | 612 |  |  | 0.223 | 0.286 | 0.0555 | 1.14 |
| 10 distance | 1616.7 | 1.05 | 0.676 | 16 | 616.7 | 1.15 | 0.577 |  | 472 | 706 | 66.1 | 3.53 | 0.109 | 0.0799 | 0.0172 | 1.69 |
| 17 strong | 21.222 .2 | 1.54 | 0.615 | 21.2 | 222.1 | 1.59 | 0.583 |  | 693 |  | 68.6 | \％ 3.18 | 0.596 | 0.446 | 0.108 | 1.39 |
| 22 size | 44.845 .3 | 5.88 | 0.08 | 44.8 | 4.9 | 5.45 | 0.211 |  | 1e＋03 | 1．36e＋03 |  | 2.74 | 0.27 | 0.314 | 0.0474 | ． 929 |

Table 2：Error of approximating real antonymic pairs（Err），mean and standard deviation $(m, \sigma)$ of error with 100 random pairings，and the ratio $r=\frac{\mid \text { Err－m｜}}{\sigma}$ for different features and embeddings

HLBL and SENNA vs 4lang
Judgments under the three given embeddings and 4lang are highly correlated，see table 3．Unsurprisingly，the strongest correlation is between the original and the scaled HLBL results． Both the original and the scaled HLBL correlate notably better with 4lang than with SENNA，making the latter the odd one out．


## Application

the dictionary－based embedding enables us to investigate the function application issue
－asymmetric expressions：john HAS dog，dog HAS john 4lang：a semantic representation in which predicates have at most two arguments
－two transformations $T_{1}$ and $T_{2}$ to regulate the linking of arguments
James kills James is agent $\quad V($ James $)+T_{1} V$（kill） kills James James is patient $V$（James）$+T_{2} V$（kill）
distinguish agent and patient relatives as in the man that killed James versus the man that James killed．

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